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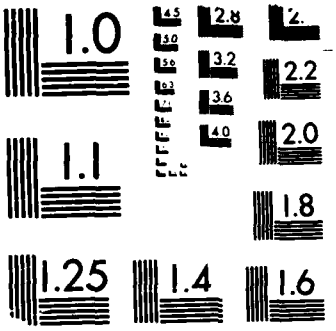

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CENTER FOR STOCHASTIC PROCESSES

Department of Statistics
University of North Carolina
Chapel Hill, North Carolina

A NOTE ON VECTOR BIMEASURES

by

Christian Houdré

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A NOTE ON VECTOR BIMEASURES

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Christian Houdré*
Center for Stochastic Processes
Department of Statistics
University of North Carolina at
Chapel Hill

Abstract A Fubini type theorem is obtained for vector bimeasure integrals.

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Morse and Transue [6-8] initiated the development of a theory of integration with respect to a bimeasure which was subsequently studied by Thomas [10]. For these authors, bimeasures are continuous bilinear functionals on $C_c(E_1) \times C_c(E_2)$, where $C_c(E_i)$, i=1,2, are the usual spaces of continuous functions with compact support on the locally compact Hausdorff spaces E_i, i=1,2. More recently, motivated by the problem of finding a Fourier representation for the covariance of a second order process, this theory has been expanded by Niemi [9] and Chang and Rao [2]. Both the bilinear functional and the set function approaches have now been studied as well as the Banach valued case developed by Ylinen [11].

In the works mentioned above the authors consistently impose, in their definition of integrability, a Fubini type condition which cannot usually be bypassed. The purpose of this note is to show that under a suitable restriction of the definition of integrability, the Fubini-type requirement becomes obsolete.

Let X be a Banach space over $\mathbb{F} = \mathbb{R}$ or $\mathbb{C}$ and let $(E,\mathcal{M})$ be a measurable space. A *vector measure* is a σ-additive set function $\mu: \mathcal{M} \longrightarrow X$. Integration of functions $f: E \longrightarrow \mathbb{F}$ with respect to vector measures is taken in the Bartle-Dunford and Schwartz [1] sense, the reader being referred to Dunford and Schwartz [3 IV 10] for the properties of this vector integral.

Let $(E_1,\mathcal{M}_1)$ and $(E_2,\mathcal{M}_2)$ be two measurable spaces. A *vector bimeasure* (bimeasure when $X = \mathbb{F}$) is a separately σ-additive set function $\beta: \mathcal{M}_1 \times \mathcal{M}_2 \longrightarrow X$, i.e., $\beta(\cdot,B)$ and $\beta(A,\cdot)$ are vector measures for all $A \in \mathcal{M}_1$, $B \in \mathcal{M}_2$.

The proof of our result as well as our definition of integrability will rely on the following two lemmas. The first one is classical and can be

found in [3 p. 323] while the second is in [11].

Lemma 1 Let $f: E \longrightarrow \mathbb{F}$ be μ-integrable. Then, the set function $\nu(A) = \int_A f d\mu$, $A \in \mathcal{M}$ is a vector measure.

Lemma 2 Let $f: E_1 \longrightarrow \mathbb{F}$ be $\beta(\cdot,B)$-integrable for all $B \in \mathcal{M}_2$. Then the set functions ${}_f\beta(A,\cdot): \mathcal{M}_2 \longrightarrow X$, $B \longrightarrow {}_f\beta(A,\cdot)(B) = \int_A f d\beta(\cdot,B)$ are vector measures for all $A \in \mathcal{M}_1$.

In the above and for $g: E_2 \longrightarrow \mathbb{F}$ the vector measures $\beta_g(\cdot,B)$ can be obtained in a completely symmetrical way.

We can now define integrability.

Definition 3 A pair of functions (f,g), $f: E_1 \longrightarrow \mathbb{F}$, $g: E_2 \longrightarrow \mathbb{F}$ is said to be integrable with respect to the vector bimeasure $\beta: \mathcal{M}_1 \times \mathcal{M}_2 \longrightarrow X$ (β-integrable for short) if the following two conditions hold.

(i) f is $\beta(\cdot,B)$-integrable for all $B \in \mathcal{M}_2$ and g is $\beta(A,\cdot)$-integrable for all $A \in \mathcal{M}_1$.

(ii) f is $\beta_g(\cdot,B)$-integrable for all $B \in \mathcal{M}_2$ and g is ${}_f\beta(A,\cdot)$-integrable for all $A \in \mathcal{M}_1$.

Remark 4 For $X = \mathbb{F}$ our definition of integrability is stronger than that of Morse and Transue. For these authors, (f,g) is integrable if in (i) and (ii) A and B are replaced by E_1 and E_2 and if in addition

$$\int_{E_1} f d\beta_g(\cdot,E_2) = \int_{E_2} g d_f\beta(E_1,\cdot). \tag{1}$$

It is also more restrictive than the *strong integral* of Niemi or the

β-integral of Ylinen. For both of them, a pair (f,g) is integrable if in (ii) A and B are respectively replaced by E_1 and E_2 and if in addition (1) is satisfied.

However, our definition is weaker than the *strict β-integral* of Chang and Rao (there is no Borel assumption on f and g or the additional Fubini *condition*).

As already mentioned, with other definitions of integrability, (1) cannot be bypassed (see [8], [11]). However, with Definition 3, this condition will always hold.

Theorem 5 Let the pair (f,g) be β-integrable, then

$$\int_A f d\beta_g(\cdot,B) = \int_B g d_f\beta(A,\cdot), \qquad \forall\, A \in \mathcal{M}_1,\ B \in \mathcal{M}_2. \tag{2}$$

The common value in (2) can thus be denoted by $\int_A \int_B f g d\beta$.

Proof Let (f,g) be β-integrable. If both f and g are simple functions, then (2) is trivial. Let f and g be bounded (f and g are measurable since integrable in the Bartle-Dunford and Schwartz sense). Then, f and g are *uniform limits of simple functions and by the dominated convergence theorem* for vector measures (see [3 p. 328]) (2) is again true.

Let f be bounded and let $B_n = \{y \in B | n \le |g| < n+1\}$.

Then, $$\int_B g d_f\beta(A,\cdot) = \sum_{n=0}^{\infty} \int_{B_n} g d_f\beta(A,\cdot) \qquad \text{(Lemma 1)}$$

$$= \sum_{n=0}^{\infty} \int_A f d\beta_g(\cdot,B_n) \qquad \text{(g is bounded on } B_n\text{)}$$

$$= \int_A f d\beta_g(\cdot,B) \qquad \text{(Lemma 2)}.$$

If f is not bounded, then $A = \bigcup_{n=0}^{\infty} A_n$ with f bounded on each A_n. Hence,

$$\int_A f d\beta_g(\cdot,B) = \sum_{n=0}^{\infty} \int_{A_n} f d\beta_g(\cdot,B) \qquad \text{(Lemma 1)}$$

$$= \sum_{n=0}^{\infty} \int_B g d_f\beta(A_n,\cdot) \qquad (f \text{ is bounded on } A_n)$$

$$= \int_B g d_f\beta(A,\cdot) \qquad \text{(Lemma 2)},$$

and the result is obtained.

Remark 6 Let $X : \mathbb{R} \longrightarrow L^2(\Omega,\mathcal{F},P)$ be a continuous V-bounded process, i.e., $X_t = \int_{\mathbb{R}} e^{itx} d\mu(x)$, $t \in \mathbb{R}$, for some vector measure $\mu: \mathcal{B}(\mathbb{R}) \longrightarrow L^2(\Omega,\mathcal{F},P)$, ($\mathcal{B}(\mathbb{R})$ is the Borel σ-algebra of $\mathbb{R}$). Then $\beta(A,B) = E\mu(A)\overline{\mu(B)}$, $A,B \in \mathcal{B}(\mathbb{R})$, is a bimeasure and $f: \mathbb{R} \longrightarrow \mathbb{C}$ is μ-integral if and only if $(f,\bar{f})$ is β-integrable (in the sense of Definition 3). Furthermore, $EX_t\bar{X}_s = \int_{\mathbb{R}}\int_{\mathbb{R}} e^{itx}e^{-isy} d\beta(x,y)$, $t, s \in \mathbb{R}$.

In view of Theorem 5 as well as the above statements, Definition 3 appears to provide (at least in a stochastic framework) the appropriate conditions for bimeasure integration. In fact, such an analysis can be extended to matrix bimeasure, as shown in Houdré [4]. The reader is also referred to Kluvánek [5] for illuminating remarks on bimeasures.

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